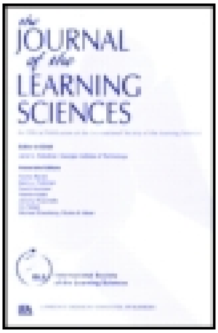

Journal of the Learning Sciences
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/hlns20

On the Science of Embodied Cognition in the 2010s: Research Questions, Appropriate Reductionism, and Testable Explanations

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Available online: 02 Sep 2011

THE JOURNAL OF THE LEARNING SCIENCES, 1–13, 2011

ISSN: 1050-8406 print / 1532-7809 online
DOI: 10.1080/10508406.2011.614325

COMMENTARY

On the Science of Embodied Cognition in the 2010s: Research Questions, Appropriate Reductionism, and Testable Explanations

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The Journal of the Learning Sciences has devoted this special issue to the study of embodied cognition (as it applies to mathematics), a topic that for several decades has gained attention in the cognitive sciences (Anderson, 2003; Barsalou; 2008; Calvo & Gomila, 2008; Lakoff & Núñez, 2000; Pfeifer & Scheier, 1999; Varela, Thompson, & Rosch, 1991; Wilson, 2002) and in mathematics education, in particular (Edwards, 2009; Nemirovsky & Ferrara, 2009; Núñez, Edwards, & Matos, 1999; Roth, 2009). The articles in this issue provide excellent examples of the type of work that needs to be produced in order to properly investigate the embodiment of mathematical cognition and this, with respect to the scope of mathematical content, the populations to be studied and the contexts in which mathematics is practiced and learned. Williams (2012/this issue) studies elementary school children learning basic mathematical concepts applied to a relevant everyday activity: clock reading and time telling. Beyond the simple understanding of numerals and numerical magnitude representing time units, his study builds on image schemas (Johnson, 1987)—a powerful concept in embodied cognition—to analyze the cognitive complexities that children need to coordinate in order to learn a seemingly banal activity. Alibali and Nathan (2012/this issue) analyze elementary and middle school teachers and students working with simple arithmetic and algebraic concepts and formulations. Focusing on gesture production,

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a topic that has gained substantial attention in embodied cognition, these authors make the case that mathematical cognition is embodied in two important senses: It is based in perception and action, and it is grounded in the physical environment. Nemirovsky, Rasmussen, Sweeney, and Wawro (2012/this issue) examine prospective secondary school teachers discussing fundamental ideas about operations with complex numbers, and they provide a fine-grained description of bodily movements and gestures coproduced with thinking and communicative processes in real-world settings. And as the guest editors Rogers Hall and Ricardo Nemirovsky (2012/this issue) point out, the underlying research designs of these studies represent approaches that conduct observations with both preexisting instructional settings (Alibali & Nathan, 2012/this issue; Williams, 2012/this issue) and with settings designed to support learning about concepts in new ways (Nemirovsky et al., 2012/this issue).

The work reported in this special issue was conceived and designed to address crucial questions in embodied mathematical cognition: How is mathematics cognition embodied (Alibali & Nathan, 2012/this issue)? What is the bodily engagement in mathematics learning (Nemirovsky et al., 2012/this issue)? What are the bodily bases of the conceptual system we use to think mathematically (Williams, 2012/this issue)? These deep and intriguing questions invoke complex cognitive phenomena such as learning, thinking, conceptual systems, bodily experience, notation systems, and mathematics. But questions, of course, don't exist in thin air. Questions can be asked—and answered—in different realms. We thus can take these questions as philosophical questions, as software design–related questions, as scientific questions, or even as literary questions. Because the goals and the practices of each of these realms are fundamentally different—none of them being ultimately better or worse per se but simply different, as they respond to different needs and goals—how we go about answering them varies substantially depending on the realm in which the questions are asked. When they are posed in the *Journal of the Learning Sciences*, however, I take them to be asked—as the title of this journal indicates—as *scientific* questions. Although the term *science* is somewhat polysemous—carrying different meanings in neuroscience, military science, or Christian science—here I safely adopt a widely accepted reading that considers *science* to be that area of human intellectual and practical endeavor that investigates the physical and the natural world (which includes humans and their activities), gathering knowledge via systematic empirical observation and experimentation. It is important to point out, of course, that not all human knowledge is scientific (why should it be?!). Knowledge in cooking, poetry, childrearing, design, politics, filmmaking, sports coaching, and many areas in education is to a great extent successfully developed outside of scientific practices. Indeed, in some of these domains the success of knowledge gathering is accomplished precisely because it is *not* scientific! But when knowledge is (or intends to be) gathered scientifically, certain conditions apply.

Science, as a human form of sense making, has three general but very clear goals, namely to *describe*, to *explain*, and to *predict* phenomena. It also has as one of its central tenets the principle of *testability* (i.e., it should be possible in principle to empirically test whether the claims and hypotheses in question hold). Contrary to what is often believed, what makes a question *scientific* is not that the answers are given based on quantitative observations (although sometimes these are helpful) or that explanations are mechanistic or deterministic. Rather, what is essential about a scientific question is that its formulation should yield answers and explanations that are testable, even if the actual realization of the testing may take decades (as is the case with many questions in theoretical physics). Thus, the question of whether God exists can be a religious or philosophical question, but it is not a scientific question because no satisfactory operational definition of "God" can lead to a conclusive test of a proposed answer. Similarly, and closer to issues in embodied cognition, the now-classic "What is it like to be a bat?" question is a philosophical question (Nagel, 1974) that has led to productive philosophical inquiry and theoretical debates in cognitive science, but it is not a scientific question. No considerations of the scientific method can provide a testable statement characterizing the actual *experience* of being a bat. Philosophical questions of this sort, however, although not themselves scientific questions, can lead to fruitful theoretical developments (e.g., focusing on new problems, refining issues, making relevant distinctions, and defining technical concepts) and to well-defined scientific questions, provided that they are operationalized in testable terms. And new empirical findings can in turn lead to new philosophical and theoretical questions, and so a mutually enriching process of understanding unfolds in an ongoing way.

Why is empirical testability important for embodied cognition? Because many academic fields under the umbrella of *humanities*—from psychology to sociology to education (mathematics education included)—have historically been notoriously weak at dealing with this dimension of systematic inquiry, being largely self-validating and sometimes even flatly tautological. The embodied cognition of the 2010s, fortunately, stands on solid and sophisticated theoretical and philosophical work. But at this point in history, it should make serious efforts in developing theories that are testable. Throughout its three decades or so of existence, embodied cognition has brought forth a diverse collection of theoretical issues ranging from the classic philosophical mind–body problem, to the phenomenology of bodily experience, to the grounding of cognitive mechanisms and their neural correlates. In embodied mathematical cognition, corresponding theoretical proposals have been advanced and claims have been made, some of which can be seen in the articles collected in this special issue (e.g., investigating thought–gesture coproduction, analyzing the bodily bases of the conceptual system used to think mathematically). After nearly three decades of embodied cognition it now seems relevant to ask to what extent questions in this theoretical framework can be addressed *scientifically*. Do we now have the appropriate theoretical and methodological developments that would make this possible? And, if yes, what are these questions? Given the advances in the study of the mind, what kinds of questions should we be asking now at the beginning of the 2010s?

A CHALLENGE FOR THE EMBODIED COGNITION OF THE 2010s: AVOIDING UNWARRANTED REDUCTIONISM WHILE REMAINING TESTABLE

Since antiquity the study of the mind has generated rich and deep questions: What is perception? How does memory work? What is the nature of language? What mental deficits are produced by head injuries? And so on. Based on countless careful descriptions by practitioners of the mind and philosophers over centuries, hundreds of thousands of pages have been written attempting to make sense of how the mind works—from the balance of four bodily humors in the Hippocratic method to the id, ego, and super-ego structure in Freudian psychoanalysis. But unlike claims about the solar system, which, since the times of Kepler around the turn of the 17th century, have been exposed to considerable empirical testing, most claims about how the mind works remained untested, protected under the realms of purely philosophical investigation, religious doctrines, and other forms of social practices. It was only in 1879, with the creation of the first laboratory of psychological research by Wilhelm Wundt in Leipzig, that some specific explanatory proposals about perception that had originated in philosophical and theoretical work started to be analyzed under the scrutiny of methodical empirical observation and testability. This date also marked the beginning of psychology as an empirical discipline separate from purely philosophical inquiry.

With respect to the more specific area of cognition, in the past century or so theories have been proposed, tested, partially falsified, rejected, and modified via the use of the scientific method. This scientific approach, however, brought with it another problem: unwarranted *reductionism*—the practice of studying a complex phenomenon in terms of simpler ones taken to stand for the former without loss of explanatory power. Unlike planetary movements, the mind is notoriously complex and highly nondeterministic in that it is multi-caused by a profusion of parameters at every possible level, from the biological bases, to the individual psychological level, to the collective sociocultural–historical level. The great challenge for the scientific study of the mind is that explanatory proposals and predictions must be extremely sensitive to an adequate calibration of the necessary reductionistic needs such that claims remain empirically testable without falling into harmful reductionism that would annihilate the very subject matter under investigation.

In the process of making the study of the mind "scientific," one approach—behaviorism—gained tremendous popularity during the first two thirds of the

20th century. This approach attempted to benefit from the clarity and objectivity of psychophysical experimental methods and their formal characterizations to explain and predict human cognition in terms of stimulus–response associations. Although convenient and precise, these approaches were eventually shown to be over-reductionistic, failing to deal with essential mental phenomena that, unfolding inside the "black box" of the mind, could not be observed through overt behavior (e.g., thinking). As a result, this approach was then quickly eclipsed in the 1970s by the so-called cognitive revolution. Inspired by the computer revolution and the Artificial Intelligence project, the cognitive revolution directed researchers' attention precisely toward this "black box," positing that cognition and the mind were best explained in terms of information processing and symbol manipulation (Gardner, 1987). This revolution brought new methods and formalizations, but, being deeply ingrained in and inspired by the computer analogy, it favored syntax over semantics and saw mental processes as chains of meaningless symbol manipulations, logical processes, and the postulation of mental modules that would carry the computations of specific forms of information processing. With it, this influential approach (still very much alive) brought another level of reductionism: the mind seen as independent of the body and its meaningful practices. The roots of embodied cognition in the 1980s, inspired by philosophical work in phenomenology (e.g., Dreyfus, 1992), theoretical developments in biology (e.g., Maturana & Varela, 1980), and advances in cognitive psychology (e.g., Rosch, 1973) and cognitive semantics (e.g., Lakoff & Johnson, 1980), among others, emerged as an attempt to remedy this fundamental problem and postulated instead the mental to be intrinsically co-defined with the body: the mind as fundamentally embodied and situated (Varela, 1989). This move addressed the reductionistic problem brought forth by the computer-driven mind/body dualism (Núñez & Freeman, 1999) but in itself, as a general theoretical proposal, did not directly provide the conditions for a testable theory of the mind. As a young new approach, the embodied cognition of the 1980s needed time to define and unfold the relevant issues that would enrich its modes of inquiry, to incorporate the development of empirically observable phenomena and testable theories. The articles we see in this special issue—manifesting a strong commitment to avoiding the use of over-reductionistic methods for investigating mathematical cognition—build on the efforts deployed since those early years of embodied cognition and lie right on the path leading toward the achievement of this goal.

DESCRIPTIONS, EXPLANATIONS, AND PREDICTIONS

Scientific disciplines exhibit different levels of advancement with respect to the developments of the descriptions, explanations, and predictions they provide. Astronomy, for instance, is in a good position for describing, explaining, and predicting with exquisite detail when and where the next solar eclipse is going to take place but not when the next supernova will be observed. Biology and medicine may be in a good position for describing the placebo effect (e.g., a harmless pill with no active ingredient produces therapeutic improvements nonetheless) but certainly not for actually explaining it. And meteorology and seismology can produce rich descriptions and elaborated explanations of weather and seismic patterns, respectively, but they are particularly bad at making long-term predictions about them. Embodied cognition, then, by no means should feel ashamed of being, say, relatively good at making descriptions of bodily movements and gestures while speech is being produced but not very good (yet) at explaining or making predictions about how exactly these resources are "spontaneously" deployed for the enactment of cognitive functions. For this to happen, clever methods and research designs are needed that can provide testable hypotheses.

Historical records show that scientific disciplines tend to achieve reasonably good levels of explanations and predictions after developing substantial bodies of descriptions. Thus, Darwin, before proposing his explanations and theoretical principles of evolution, needed rich descriptions of the diversity of living beings, their behaviors, and the ecological niches they inhabit, which he observed first hand at remote places overseas. Similarly, in the early 1980s, before being able to propose explanations of the causes of AIDS, the community had to embark on massive-scale descriptions of behavioral, epidemiological, cultural, sociological, and biochemical patterns observed among thousands of patients across the world.

Descriptions are crucial in the scientific method, but by themselves they do not constitute the ultimate goal of the scientific enterprise. Descriptions provide the ground for the generation of testable hypotheses, and thus they are needed for the elaboration of explanations and predictions. In its three decades or so of existence, the investigation of embodied cognition has not been an exception in this regard. The field started with the help of rich descriptions of phenomenological experiences that led to a variety of theoretical considerations and detailed characterizations suggesting that the mind–body system is not reducible to disjointed parts where cognition is concerned (Varela et al., 1991). And insightful philosophical arguments were elaborated (Johnson, 1987), describing, for instance, how everyday bodily grounded experience underlies semantic and inferential structure that can be observed in linguistic metaphorical expressions (Lakoff & Johnson, 1980), which were claimed to provide structure to *conceptual* systems and high-level *reasoning*. All this intellectual work was primarily philosophical and theoretical. But by the 1990s, terms such as *conceptual* and *reasoning* had an established tradition of empirical work in psychology such that claims of this sort could then be studied empirically with newly developed methods in psycholinguistics. Psychologists, empirically oriented cognitive linguists, and other cognitive scientists began to test these claims experimentally (e.g., Gibbs, 1994), confirming some of them and disproving others. Nowadays, in conceptual metaphor theory, for example, an extraordinary number of studies using various methods, from psycholinguistic experiments, to gesture studies, to electroencephalography, to functional magnetic resonance imagery, have been carried out to empirically investigate many claims regarding the embodiment of conceptual metaphor and its psychocognitive reality (for a review, see Gibbs, 2008). What can then be said of embodied mathematical cognition? Has this subfield reached a level of development and maturity such that theoretical and philosophically inspired claims can now be tested empirically?

EMBODIED MATHEMATICAL COGNITION, METHODS, AND TESTABILITY

Developments in embodied mathematical cognition face different types of difficulties and lie at various levels of complexity such that empirical testability is not always straightforward. My own work in mathematical cognition, for instance, has in some respects gone through a path similar to the one described previously, beginning with philosophical considerations (Núñez, 1995, 1997) and theoretical and descriptive work (Lakoff & Núñez, 2000; Núñez & Lakoff, 1998)—not directly testable (or definable in adequate operational terms)—followed (in some domains) by a gradual and partial move toward the consolidation of testable claims whenever theoretical and methodological advancements permit it. In *Where Mathematics Comes From* (2000), for example, based primarily on theoretical work driven by results in embodied cognitive semantics, Lakoff and I made many claims and hypotheses about the nature of mathematics and mathematical cognition. Some of them pertained to the realm of philosophy (e.g., our claim that "Platonism is inadequate for characterizing the nature of mathematics"). Others were historical but not directly testable (e.g., what the mathematician George Cantor "had in mind" when conceiving his notion of transfinite cardinals, for which the closest "evidence" comes from Cantor's own writings). And some were claims that eventually ended up being operationalized in testable terms with the goal of explaining and predicting (not just describing) specific scientific questions. We asked, for instance, why is it that when the concepts of limits and continuity of functions are introduced in classic mathematics books they do not express directly the ideas underlying the usual static ε-δ formalisms when these introductions were meant to be precise and rigorous? Why do they exhibit instead dynamic language, as in "the function *oscillates* more and more"? Our theoretical cognitive semantic analysis predicted that the precise meaning of limits and continuity of functions is, contrary to a widespread belief, provided not by the rigor of the usual static ε-δ formalisms (which involve existential and universal quantifiers) but by a set of everyday embodied cognitive mechanisms with precise inferential structure dealing with imaginary dynamic entities conceived via conceptual metaphor, source–path–goal schemas (Lakoff & Johnson, 1980; Lakoff & Núñez, 2000), and fictive motion (Talmy, 2000). In this example, the empirical test of our hypothesis came some years later in the form of largely unconscious and unmonitored motor action of the type analyzed by Alibali and Nathan in this special issue: speech–gesture coproduction. The co-speech gestures produced by professional mathematicians teaching university-level calculus classes did not match the meaning and inferential structure of the ε-δ formalisms (despite the fact that they were overwhelmingly present in written form on the blackboard) but rather—as predicted—they matched the dynamic meaning evoked via the underlying conceptual metaphors (e.g., NUMBERS ARE LOCATIONS IN SPACE), source–path–goal schemas, and fictive motion (Núñez, 2006). Similar evidence was obtained in a semicontrolled experimental setting through gestures produced by mathematics graduate students working in dyads while proving a specific theorem in analysis (Marghetis & Núñez, 2010), confirming that these dynamic gestures are not only produced in a "teaching mode" but that they are also produced in a "working mode" when communicating with fellow mathematicians. Many of the claims about embodied mathematical cognition in *Where Mathematics Comes From* remain theoretically plausible but for the moment empirically untested. If we want such work to make progress in the realm of scientific inquiry (which, again, one could decide not to do and choose instead to keep in the realms of philosophical or theoretical inquiries), we then must move it along to meet the demands of empirical testability.

The papers included in this special issue present clear attempts to put the study of embodied mathematical cognition on the empirical map—without falling in the over-reductionistic trap, by investigating mathematical cognition in relevant observable bodily related terms: rigorous experiments with speech–gesture coproduction (Alibali & Nathan), analyses of bodily movements in communication of mathematical content (Nemirovsky et al.), studies of image-schematic structure underlying inferential processes (Williams). All three papers clearly endorse non-reductionistic approaches to studying the learning mind by conducting investigations through ecologically valid methods: analyzing reasoning and communication in real time, gesture coproduced with verbalizations and bodily movements, and the opportunistic use of the environment for unfolding thinking processes in the real world. These valuable empirical observations are largely possible today thanks to new digital audio/video technologies that were not readily available some years ago. Carrying out similar observations and data analysis in the 1980s, at the beginning of the embodied cognition movement, would have been cumbersome and expensive, and in the 1960s or 1970s it would have been nearly impossible. In other words, claims about, say, the tight synchronicity of human speech, gesture, eye movements, prosody, and thought were simply not available for philosophers and psychologists at the turn of the 20th century. Phenomenologists of the time had to develop their ideas and theories based on their coarse-grained observations, intuitions, and introspections. What would have been the phenomenology of Husserl, Heidegger, and Merleau-Ponty if their starting point had been the fine-grained observations we have today? Nowadays, digital technology provides new "microscopes" and "telescopes" that take the observation of the embodied mind to a new level, revealing phenomena that were never before available in the history of humankind. The papers in this special issue, as well as others in the learning sciences (Derry et al., 2010), reflect this seemingly trivial but crucial fact.

With respect to the scientific criteria of the questions and their answers, the papers in this special issue take different positions. The gesture production analyzed by Alibali and Nathan and the image-schema analysis presented by Williams endorse a clear commitment to testability in that beyond the descriptions they provide, their explanations and predictions are testable according to specified standards. In this sense, their research questions—"How is mathematics cognition embodied?" and "What are the bodily bases of the conceptual system we use to think mathematically?" respectively—can be taken as scientific questions. The paper by Nemirovsky et al., however, defends a contrasting approach in this regard, which results in a rather different reading of their research question: "What is the bodily engagement in mathematics learning?" There is no doubt that in addressing this question Nemirovsky et al. make a clear commitment with empirical observations (e.g., microethnography, gesture analysis), but their methods and reasoning appear to depart from an approach that endorses testability. For instance, these authors attempt to investigate two conjectures regarding mathematical insights and ideas. And they argue that a phenomenological approach is needed to do so, one that makes use of the Husserlian notion of "horizon," which, when applied to perceptuo-motor activity, yields a "realm of possibilities" enacted by mathematics learners. This move, they argue, should cope with the "explanatory gap" between "the mechanistic/mathematical accounts of a phenomenon and the subjective qualities lived by the participants" (p. XX). Although this is reasonable—in fact, essential—in a phenomenological inquiry, it gets in the way of providing a scientific explanation, because descriptions in terms of "realms of possibilities" and interpretations of "lived experience" lack standards for testability. *Experience* as such, as we saw earlier, is not testable in itself. Pain is pain, which is different from an (accurate or inaccurate) *explanation* of the pain. Explanations of *why* pain occurs, however—although not being pain itself—are testable, and this is independent of the models being mechanistic or deterministic. Indeed, in another area of embodied cognition—the development of motor action—Thelen and Smith (1994) provided compelling (testable) explanations of how children learn to coordinate movement, action, and cognition through nondeterministic, nonmechanistic, and nonlinear models. The microethnography of Nemirovsky et al.'s case studies provide rich and insightful descriptions of gestures and bodily action while learning and discussing mathematical ideas. If, in the context of research in the *learning sciences*, the goal is to describe, explain, and predict bodily engagement in mathematics learning, it is unclear why this needs a phenomenological approach. In fact, Nemirovsky et al.'s study seems to have all the ingredients for providing rich and fruitful testable hypotheses that can make notable contributions to embodied mathematical cognition. For instance, based on their observations, semicontrolled empirical observations could be further elaborated in order to ask the following: What are the cognitive mechanisms and the construals underlying the very "realms of possibilities" that mathematics learners (and professional mathematicians) enact in different situations? Why are certain "realms of possibilities" and not others brought forth in certain specific conditions? And, is it possible to make predictions about what "realms of possibilities" are going to be enacted by mathematics learners when immersed in different (controlled) settings and presented with certain types of materials, or in certain social contexts and not others? And so on. The rich and subtle descriptions are there. It seems that now is the moment for elaborating possible explanations and predictions that can be tested systematically.

CHALLENGES TO EMBODIED MATHEMATICAL COGNITION

Many questions—scientific questions, that is—remain, of course, open in the study of embodied mathematical cognition. First, Alibali and Nathan (2012/this issue) argue that mathematical cognition is embodied in two senses: It is based in perception and action, and it is grounded in the physical environment. But if so, a crucial question is then why other primates with bodies; with bodily experience of space, gravity, and motion; and that exhibit social behavior, emotions, and memories that afford similar perception and actions and grounding in physical environments *do not* develop mathematical concepts (or symbolic language with syntactic structures, for that matter). In this sense, Williams's (2012/this issue, p. XX) question is essential: "What are the bodily bases of the conceptual system we use to think mathematically?" His question implies that just having a body and the related bodily experiences—perception and action mechanisms, and grounding in the physical environment—is not what gives us mathematics and mathematical ideas but that the conceptual system that sustains mathematical thinking may be uniquely human, having bodily bases that need to be investigated via explanatory proposals that can be established in testable terms. Second, another crucial and related question is, how do we develop mathematical concepts about entities that by definition we cannot experience through our bodily senses, such as actual infinity? Alibali and Nathan cite influential work on embodied cognition by Barsalou (2008) and Wilson (2002), who argued that human cognitive and linguistic processes are rooted in perceptual and physical interactions of the human body with the world. Although this view of embodied cognition may be appropriate when the concepts under investigation are "chairs" and "balls," it is not clear how this view would give a full account of cases in which the invoked entities *by definition* cannot be perceived directly through the senses: "the point at infinity" in projective geometry or "the Euclidean point," which has only location but no extension. If embodied mathematical cognition is primarily about perceptual and physical interactions with the real world, how then can these "simple" mathematical notions be "embodied" if no such interaction can exist? And if, as Alibali and Nathan argue, embodied cognition works in terms of simulations of action and perception, how can we simulate actions and perceptions of entities to which we do not have access through experience? Encapsulating many important challenges for the embodiment of high-level cognition (Núñez, 2008), infinity seems to prove that mathematical ideas are de facto *dis*-embodied, because no bodily-grounded experience sustained by our finite brains and bodies can provide the necessary basis for such notions. But if we understand that embodied mathematical cognition (and presumably embodied cognition in general) is not *just* about direct bodily experience, perceptuo-motor activity, direct simulations of action and perception, and physiological mechanisms per se but, as Williams argues, about the *bodily basis of the conceptual system* we deploy to think mathematically (e.g., image schemas and conceptual mappings), then we can attempt to provide testable explanations of the nature of mathematical infinities in terms of human cognition and its bodily basis. Fortunately, as the papers in this special issue show, we now have developing empirical methods that give us the unique opportunity to ask these questions in the realm of science. Deep philosophical insights about the body–mind and the ethnography of human behavior have provided solid ground for a rich theoretical corpus on embodied cognition. Now, at the beginning of the 2010s, and supported by new technologies and methods, the time is ripe for embodied mathematical cognition to move toward paradigms and theories that can be empirically testable. These are exciting times indeed!

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