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The language of mathematics: Investigating the ways language counts for children's mathematical development

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This longitudinal study examined how language ability relates to mathematical development in a linguistically and ethnically diverse sample of children from 6 to 9 years of age. Study participants were 75 native English speakers and 92 language minority learners followed from first to fourth grades. Autoregression in a structural equation modeling (SEM) framework was used to evaluate the relation between children's language ability and gains in different domains of mathematical cognition (i.e., arithmetic, data analysis/probability, algebra, and geometry). The results showed that language ability predicts gains in data analysis/probability and geometry, but not in arithmetic or algebra, after controlling for visual–spatial working memory, reading ability, and sex. The effect of language on gains in mathematical cognition did not differ between language minority learners and native English speakers. These findings suggest that language influences how children make meaning of mathematics but is not involved in complex arithmetical procedures whether presented with Arabic symbols as in arithmetic or with abstract symbols as in algebraic reasoning. The findings further indicate that early language experiences are important for later mathematical development regardless of language background, denoting the need for intensive and targeted language opportunities for language minority and native English learners to develop mathematical concepts and representations.

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Introduction

There is growing recognition that language ability is important for children's mathematical development (e.g., Carey, 2004; Kleemans, Segers, & Verhoeven, 2011; LeFevre et al., 2010). Indeed, Moses and Cobb (2001) argued that language underlies children's ability to gain the conceptual understanding necessary to make sense of the otherwise abstract symbols inherent in mathematics. It has even been suggested that children's mathematical difficulties may reflect deficient linguistic processes as opposed to deficits in nonverbal numerical processes (Lager, 2006; LeFevre et al., 2010; Vukovic, 2012). These suppositions are in part supported by neuropsychological evidence showing that the left angular gyrus supports the manipulation of numbers in verbal form (Dehaene, Piazza, Pinel, & Cohen, 2003). Yet very few studies have systematically examined the relation between linguistic processes and mathematical cognition beyond number and arithmetic.

To begin to increase the specificity of our understanding of the linguistic basis of mathematics, this study examined how language ability relates to children's development across different domains of mathematical cognition from first to fourth grades. We focused specifically on linguistically and ethnically diverse children attending high-poverty urban schools, and this represents a significant step toward advancing the science of children's mathematical development in light of trends in the demographics of school-age populations. There is a particular need for research with language minority learners—students who come from homes where the primary language spoken is not the societal language. In industrialized countries worldwide, the population of children growing up in linguistically diverse homes is on the rise (UNICEF Innocenti Research Centre, 2009). In the United States, for example, the past several decades have seen a dramatic increase in the number of school-age children coming from homes where English is not the primary language spoken; between 1980 and 2009, this population of children rose from 10% to 21% of school-age children (Aud et al., 2011). The research that has been conducted with language minority students focuses predominantly on their literacy development and its instruction. Investigating the linguistic basis of mathematics with language minority learners not only provides an important window into the relation between language and mathematics but also has implications for identifying sources of individual differences in mathematical development in this understudied population.

Role of language in mathematical cognition

The bulk of studies investigating the linguistic basis of mathematics have focused specifically on understanding whether numerical cognition represents an invented construction based on language (e.g., Le Corre & Carey, 2007; Sarnecka & Carey, 2008; Sarnecka & Lee, 2009) or whether concept of number exists independent of language (Ansari et al., 2003; Frank, Everett, Fedorenko, & Gibson, 2008; Gelman & Butterworth, 2005; Libertus & Brannon, 2010; Libertus, Feigenson, & Halberda, 2011). It turns out that language—having unique words for exact quantities specifically—plays a role in some, but not all, aspects of numerical cognition. In particular, having number words appears to be involved in the uniquely human ability to cognitively represent large numbers (i.e., $\geqslant 5$) with precision (e.g., Dehaene, Spelke, Pinel, Stanescu, & Tsivkin, 1999; Gordon, 2004; Spaepen, Coppola, Spelke, Carey, & Goldin-Meadow, 2011). This is in contrast to the preverbal number system not unique to humans that is specialized for representing only small numerosities (i.e., 1–4) precisely and large quantities approximately (see Gelman & Butterworth, 2005; Landerl, Fussenegger, Moll, & Willburger, 2009). Thus, this body of research indicates that although concepts of quantity exist independent of language, exact representations of number—which is foundational for formal mathematics—are dependent on a language system.

There is burgeoning evidence that language processes are indeed involved in solving basic mathematical problems, in particular arithmetical addition and subtraction. For instance, Russian–English bilingual adults have been shown to retrieve newly learned addition and subtraction facts more efficiently in the language of training, whether Russian or English, compared with the untrained language, suggesting that arithmetic facts are stored in language-specific ways (Dehaene et al., 1999; Spelke & Tsivkin, 2001). Indeed, Spelke and Tsivkin (2001) showed that even incidental exposure to exact numbers (e.g., learning a date in history) is stored in language-specific ways, such that language of training affects how numerical information is stored and subsequently retrieved.

It is hypothesized that phonological processes specifically underlie this relation, presumably because completing arithmetic problems requires the retrieval of phonological codes (Fuchs et al., 2005; Hecht, Torgesen, Wagner, & Rashotte, 2001; Koponen, Aunola, Ahonen, & Nurmi, 2007; Simmons & Singleton, 2008). Indeed, the well-documented relation between phonological processing and arithmetic performance helps to explain the finding that many children with reading difficulties also have difficulty with arithmetic (e.g., Dirks, Spyer, van Lieshout, & de Sonneville, 2008; Rubinsten, 2009; Simmons & Singleton, 2008). However, there exist some children with mathematical difficulties who are nonetheless good readers (and therefore tend to have age-appropriate phonological skills) and vice versa (e.g., Landerl et al., 2009; Vukovic, 2012), suggesting that phonological skills are not the sole influential factor for some children. Moreover, Jordan and colleagues have found that children can compensate for arithmetical difficulties by using verbal strategies to complete arithmetic problems, suggesting that language skills beyond phonological processing are involved in arithmetic performance (e.g., Jordan, 2007; Jordan & Hanich, 2003). There is further evidence that the language used in arithmetic problems influences how children symbolically represent and solve such problems (e.g., Abedi & Lord, 2001; Brissiaud & Sander, 2010; Lager, 2006). Together, these findings suggest that language ability more broadly may play a unique role in children's mathematical cognition.

Yet the role of language in more abstract mathematical concepts has been relatively unexplored, especially in more diverse samples of children. In a relevant study, LeFevre and colleagues (2010) proposed that the linguistic circuit advanced by Dehaene and colleagues (2003) is engaged when children perform mathematical tasks that depend on the formal number system. The authors found that a linguistic composite (i.e., vocabulary, phonological awareness, and number identification) measured in a sample of 182 preschool children explained unique variance in second-grade arithmetic, numeration, geometry, and measurement. The linguistic composite was consistently the most important predictor across different domains of mathematical cognition—more so than either nonverbal subitizing or nonverbal visual–spatial working memory—suggesting that language skills play a critical role in children's understanding of not only arithmetic but also higher order mathematical domains.

That the linguistic composite used by LeFevre and colleagues (2010) included vocabulary, elision, and number identification makes it difficult to definitively conclude that language per se drives the relation between the linguistic pathway and mathematical development. Specifically, the relation between the linguistic composite and mathematical outcomes could reflect a relation between mathematical outcomes and phonological skills and/or domain-specific skills, consistent with previous research (e.g., Fuchs et al., 2005; Jordan & Levine, 2009; Locuniak & Jordan, 2008; Swanson & Beebe-Frankenberger, 2004). Indeed, Dehaene and colleagues (1999) found that whereas adults store *exact* arithmetic sums as language-based representations, *approximate* calculations—including advanced mathematical facts such as cube roots—may be completed independently of language. Therefore, the authors speculated that higher order forms of mathematics might not be as dependent on language as is arithmetic. One purpose of the current study, thus, was to build on and advance previous research (e.g., Dehaene et al., 1999; LeFevre et al., 2010; Spaepen et al., 2011) by focusing specifically on whether general language ability is related to the development of both arithmetic and higher order mathematical domains. A second purpose was to examine the role language plays in the mathematical development of language minority learners.

Linguistic diversity and mathematical development

Most research examining children's mathematical development has been conducted with native English speakers, primarily in middle-income settings, resulting in little knowledge about language minority learners' mathematical development. Studying mathematical development in language minority learners provides an important alternative angle by which to examine questions surrounding the relation between language and mathematics. Children from language minority backgrounds—especially those in high-poverty urban settings—disproportionately struggle with mathematics (Kieffer, Lesaux, Rivera, & Francis, 2009; National Center for Education Statistics [NCES], 2009), suggesting that language proficiency plays a role in mathematical development. However, these findings do not shed light on whether language plays a similar role in mathematical development in language minority learners and their native English-speaking peers or whether language proficiency serves as a barrier to mathematical performance in language minority learners simply because these children do not understand the task requirements. Indeed, in mathematics classrooms and curricula across the United States, language minority learners do not understand much of the language that is used, and most learners are not explicitly taught to read, write, or speak mathematically (Lager, 2006).

At the same time, language minority learners in the United States tend to be concentrated in high-poverty urban schools (e.g., Gándara, Rumberger, Maxwell-Jolly, & Callahan, 2003), and there is a well-established relation between poverty and underdeveloped language and vocabulary skills, at least for monolinguals (e.g., Hart & Risley, 1995). Thus, it is possible that poverty—manifested as fewer opportunities to learn in and out of school than in middle-class neighborhoods—detrimentally affects the language development of both language minority learners and their native English-speaking peers, which in turn affects the formation of concepts—including mathematical concepts—that are necessary for school success (Carey, 2004). Indeed, Lager (2006) found that many of the language obstacles that affected language minority learners' ability to successfully complete algebra word problems were also barriers for native speakers. For instance, both groups struggled to understand specialized mathematical language (e.g., pattern, show, figure), symbols and notation (e.g., parentheses), and the use of variables (e.g., "if $x = 10$, $y = ?$"). Thus, in the context of research with a diverse sample, comparing language minority learners with their native English-speaking classmates, who attend the same schools and often share many educational experiences as well as socioeconomic characteristics, offers additional insight into the role language plays in mathematical development.

The current study

This study sought to contribute to an understanding of the linguistic basis of mathematics by using a developmental lens to examine the relation between general language ability and mathematical cognition in a linguistically and ethnically diverse sample of children. Building on previous research (e.g., Dehaene et al., 2003; Kleemans et al., 2011; Le Corre & Carey, 2007; LeFevre et al., 2010), we examined whether language ability predicted gains in children's arithmetic, data analysis/probability, algebra, and geometry. Given that longitudinal studies of school-age children are largely absent from the research base, we studied the period from early to middle childhood, following a sample of children from first to fourth grades. Two research questions guided this study:

1. Does children's early language ability predict later gains in arithmetic, data analysis/probability, algebra, and geometry?
2. Do the relations between early language ability and gains in mathematical cognition differ between language minority learners and native English speakers?

Method

Participants

The data in this study were collected as part of a prospective longitudinal research project designed to examine the developmental course and cognitive and linguistic predictors of various mathematical abilities in a cohort of children in an urban context. The study participants were 167 linguistically and ethnically diverse children attending two elementary schools in a large urban center in the northeastern United States. The two schools reported using the same inquiry-based mathematics curriculum that involves little teacher-directed instruction (Everyday Mathematics; University of Chicago School Mathematics Project, 2007). Instead, children are encouraged to independently solve real-world mathematical problems through hands-on exploration and student-centered activities (for further description, see http://everydaymath.uchicago.edu). All children were taught mathematics in English.

Children were originally recruited in first grade (mean age = 6 years 10 months, *SD* = 5 months) and followed through second grade (mean age = 7 years 10 months, *SD* = 6 months), third grade (mean age = 8 years 11 months, (*SD* = 6 months), and fourth grade (mean age = 9 years 11 months, *SD* = 6 months). Approximately half of the participants were female ($n = 81$, 48.5%) and 96.8% of the children received free or reduced price lunch. Of the 167 children, 75 (44.9%) were native English speakers, 92 (55.1%) were language minority learners (Spanish speaking $n = 78$ and other $n = 14$). The majority (60.0%) of native English speakers were Black, 33.3% were Hispanic, and 6.7% were "other" or did not report race/ethnicity. The majority of language minority learners (87%) were Hispanic, 5.4% were Black, and 7.6% were other or did not report race/ethnicity. There were no differences between native English speakers and language minority learners in the percentage of students who qualified for free or reduced price lunch (95.5% of native English speakers and 97.7% of language minority learners).

There was some attrition from the cohort over time. Of the children originally recruited in first grade, 15 children (9.0% of the total sample) did not participate in later grades, whereas an additional 2 children (1.2%) participated in Grade 2 but not in later grades and 1 child (0.6%) participated in Grade 3 but not in Grade 4. In turn, the sample was refreshed in each year of the study, with 38 children (22.8%) joining the study in second grade, 7 children (4.2%) joining in third grade, and 13 (7.8%) joining in fourth grade. Excluding cases with incomplete data across the four testing occasions could lead to substantial bias in the estimates as well as reduced statistical power. Thus, all children who participated in at least one testing occasion were included in the analytic sample, and full information maximum likelihood (FIML) was used to account for missing data on the language, mathematics, and control variables (there were no missing data for language status or sex). In FIML estimation, all available information is used to estimate the variance–covariance matrix, which then serves as the basis for fitting structural equation models; this approach has been shown recently in Monte Carlo studies to yield equal or superior results to other approaches to missing data (e.g., Enders & Bandalos, 2001).

Measures

Language ability

Both Dehaene and colleagues (2003) and LeFevre and colleagues (2010) hypothesized that the linguistic basis of mathematics stems from a general language system as opposed to a specialized mathematical language system. As such, we selected two indicators of general language ability: vocabulary and listening comprehension. Consistent with LeFevre and colleagues, we hypothesized that general vocabulary also reflects children's ability to acquire vocabulary in the number system. We used the Picture Vocabulary test from the Woodcock–Johnson III (WJ-III; Woodcock, McGrew, Schrank, & Mather, 2007) to index vocabulary. With Picture Vocabulary, children identify pictured objects ranging from common to specialized (e.g., star, gavel). The publisher reports reliability between .70 and .77.

We hypothesized that the general ability to listen and comprehend oral communication would also reflect children's ability to comprehend mathematical content. We used the WJ-III Oral Comprehension test (Woodcock et al., 2007). With Oral Comprehension, children listen to short passages and then supply the missing final word using syntactic and semantic cues (e.g., "Without a doubt, his novels are more complex than the novels of many other contemporary ____"). The publisher reports reliability between .78 and .83. As described below in preliminary analyses, we created a language ability latent composite for these two measures to use in subsequent analyses.

Arithmetic

The Computation subtest of the Stanford Diagnostic Math Test–Fourth Edition (SDMT-4; Harcourt Brace Educational Measurement, 1996) was used to measure the degree to which children have mastered basic number facts and are able to use procedural computational skills to solve arithmetic problems with the four operations, including problems requiring regrouping. With this test, children have 25 min to complete 20 grade-level questions presented in arithmetic notation. The publisher reports Kuder–Richardson Formula 20 reliability of .84 or .85.

Higher order mathematical domains

We used the KeyMath Diagnostic Assessment–Third Edition (KeyMath-3; Connolly, 2007) to assess three higher order domains of mathematical cognition: data analysis/probability, algebra, and geometry. With the Data Analysis/Probability subtest, children interpret tables (e.g., "Here is a table that shows the eye color of all the children in a classroom. How many children have blue eyes?"), interpret tally charts (e.g., "Here is a tally of the animals in a zoo. Altogether, how many gorillas and giraffes are there?"), and estimate probability (e.g., "If you want to land on purple, which spinner should you choose?"); the publisher reports reliability between .75 and .84. With the Algebra subtest, children work with number sentences (e.g., "Six plus some number equals ten. Point to the missing number."), describe patterns and functions (e.g., "If each circle stands for the same number, and four circles equals 12, what number does one circle stand for?"), and represent mathematical relations (e.g., "Eight equals six plus what number?"); the publisher reports reliability between .80 and .83. With the Geometry subtest, children analyze, describe, compare, and classify two- and three-dimensional shapes (e.g., "Which shape doesn't belong in this group?"), solve problems involving the relation between two-dimensional and three-dimensional objects (e.g., "Which puzzle piece will fit into this hole?"), and use visualization and formulas to solve problems (e.g., "Which shape will have the most blue squares when filled completely?"); the publisher reports reliability between .74 and .81.

Control variables

In each set of analyses, we included a set of control variables to provide for a more stringent test of our models. We used the WJ-III Research Edition (Woodcock, McGrew, & Mather, 1999) Letter–Word Identification test to control for general achievement. We used the Swanson Cognitive Processing Test (S-CPT) Visual Matrix subtest (Swanson, 1996) to control for visual–spatial working memory because it has been hypothesized to represent a distinct pathway to mathematics separate from a linguistic pathway (e.g., Dehaene et al., 2003; LeFevre et al., 2010). Finally, consistent with previous research examining influences on mathematical cognition (e.g., Jordan, Hanich, & Kaplan, 2003; LeFevre et al., 2010), we also controlled for sex.

Procedure

Annual assessments of children's skills occurred from January to March. Children were assessed individually for all tasks except SDMT-4 Computation, which was group administered in children's classrooms. All tasks were administered in English. SDMT-4 Computation and KeyMath-3 Data Analysis/Probability were administered from first to fourth grades, whereas KeyMath-3 Algebra and KeyMath-3 Geometry were administered from second to fourth grades. Children completed the paper-and-pencil SDMT-4 Computation questions on their own. By contrast, children were presented with each item from the KeyMath-3 subtests one at a time, with children seeing the stimulus on their page (e.g., tally chart, unfinished number sentence, incomplete shape) and the examiner identifying the problem to be solved (e.g., "How many gorillas and giraffes are there?" "Six plus some number equals ten. Point to the missing number." "Which puzzle piece will fit into this hole?"). In other words, the children were not simply solving word problems, nor did any of the mathematical tasks have significant reading demands. Research assistants conducted the assessments in the schools. Research assistants completed an intensive 4-h training workshop on standardized administration, which included demonstrating 100% accuracy during mock administrations. In addition, throughout the data collection, a school psychology doctoral student was available to provide coaching as needed.

Results

Preliminary analyses

Prior to fitting regression models to address our research questions, we conducted several preliminary analyses, including estimating descriptive statistics, comparing language minority learners and native English speakers on all measures, and conducting exploratory analyses to assess the appropriateness of using a latent variable to represent general language ability. First, we estimated means on all of the mathematics, language, and control measures for the entire sample and separately by language group. The left side of Table 1 presents sample means in percentile ranks to facilitate

Table 1
Means for mathematics measures, language measures, and control measures overall and by language group.

			Overall (N = 167)	Language minority (n = 92)	Native English (n = 75)	Mean difference in Cohen's d (positive values favor language minority)	z-Statistic for mean difference
Mathematics							
Arithmetic	Grade 1	RS[a]	9.68	9.85	9.56	0.06	0.45
		PR	28.15	28.94	28.50		
	Grade 2	RS[a]	13.95	14.35	13.49	0.18	0.92
		PR	31.02	32.48	29.15		
	Grade 3	RS[a]	13.66	14.32	12.78	0.24	1.65
		PR	33.29	36.96	28.34		
	Grade 4	RS[a]	13.08	12.90	13.33	−0.09	0.04
		PR	29.31	28.61	30.55		
Data analysis/ probability	Grade 1	RS	5.32	4.72	6.26	−0.51	−3.04[**]
		PR	25.44	20.46	33.09		
	Grade 2	RS	7.93	7.53	8.43	−0.24	−1.08
		PR	27.42	25.59	29.84		
	Grade 3	RS	14.34	14.41	14.30	0.03	0.17
		PR	42.22	42.06	42.76		
	Grade 4	RS	16.69	16.22	17.28	−0.20	−0.94
		PR	38.89	37.88	39.86		
Algebra	Grade 2	RS	7.84	7.82	7.87	−0.01	0.05
		PR	35.46	35.28	35.87		
	Grade 3	RS	11.74	11.92	11.52	0.11	0.71
		PR	35.38	35.31	35.59		
	Grade 4	RS	14.84	14.51	15.21	−0.16	−0.77
		PR	37.06	35.1	39.08		
Geometry	Grade 2	RS	10.11	10.3	9.92	0.11	0.63
		PR	24.64	25.04	24.46		
	Grade	RS	15.38	15.6	15.21	0.12	0.58

(*continued on next page*)

Table 1 (*continued*)

			Overall ($N = 167$)	Language minority ($n = 92$)	Native English ($n = 75$)	Mean difference in Cohen's d (positive values favor language minority)	z-Statistic for mean difference
	3						
		PR	38.14	39.07	37.65		
	Grade 4	RS	17.09	17.27	16.93	0.09	0.59
		PR	34.15	35.41	32.82		
Language ability							
Vocabulary	Grade 1	RS	15.08	13.85	16.65	−0.80	−4.58***
		PR	22.93	15.96	31.86		
	Grade 2	RS	16.91	16.05	17.93	−0.66	−3.82***
		PR	22.20	17.61	27.84		
Oral comprehension	Grade 1	RS	9.27	7.93	10.81	−0.65	−2.84**
		PR	32.25	24.41	42.18		
	Grade 2	RS	10.97	9.95	12.20	−0.59	−3.26**
		PR	28.04	23.36	33.73		
Controls							
Letter–word identification	Grade 1	RS	31.53	30.7	33.23	−0.30	−1.91
		PR	65.53	63.9	69.73		
	Grade 2	RS	40.25	39.58	41.13	−0.19	−1.10
		PR	60.14	57.94	63.14		
Visual–spatial working memory	Grade 1	RS	1.27	1.31	1.23	0.06	0.01
		PR	17.87	18.15	17.62		
	Grade 2	RS	1.78	1.69	1.90	−0.18	−0.82
		PR	25.33	24.27	26.65		

Note. RS, raw score; PR, percentile rank.

[a] Forms of the arithmetic measure change over time, so raw scores are not vertically equated. Vertically equated scaled scores were used in all subsequent analyses.

** $p < .01$.

*** $p < .001$.

Table 2
Correlations among mathematics, language, and control variables for all students, with strong correlations between language and mathematics measures bolded and underlined and with strong correlations between mathematics and control measures bolded ($N = 167$).

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
1. G1 arithmetic[a]																							
2. G2 arithmetic[a]	.48																						
3. G3 arithmetic[a]	.28	.50																					
4. G4 arithmetic[a]	.29	.53	.56																				
5. G1 data analysis/probability	.44	.30	.27	.30																			
6. G2 data analysis/probability	.48	.45	.30	.42	.49																		
7. G3 data analysis/probability	.54	.51	.58	.46	.52	.55																	
8. G4 data analysis/probability	.53	.53	.39	.57	.49	.62	.60																
9. G2 algebra	.53	.57	.47	.57	.46	.66	.62	.65															
10. G3 algebra	.55	.60	.62	.50	.53	.58	.70	.59	.62														
11. G4 algebra	.44	.51	.44	.62	.42	.59	.53	.71	.67	.61													
12. G2 geometry	.43	.40	.37	.37	.37	.53	.49	.53	.55	.43	.44												
13. G3 geometry	.53	.36	.32	.30	.40	.41	.60	.47	.53	.50	.47	.53											
14. G4 geometry	.42	.37	.28	.35	.23	.52	.49	.57	.55	.43	.58	.54	.49										
15. G1 vocabulary	.35	.30	.19	.27	.44	.38	.37	.41	.47	.32	.45	.43	.41	**.52**									
16. G2 vocabulary	.29	.38	.16	.23	.48	.44	.40	.40	**.51**	.40	.43	.43	.32	.35	.73								
17. G1 oral comprehension	.36	.33	.17	.33	**.54**	**.51**	.40	**.60**	.46	.31	**.50**	.44	.35	.45	.67	.57							
18. G2 oral comprehension	.24	.30	.22	.26	**.51**	**.50**	.47	**.51**	.46	.42	.44	.44	.33	.44	.66	.61	.69						
19. G1 letter–word identification	**.51**	**.52**	.46	.44	**.50**	.43	**.51**	**.55**	**.53**	**.53**	**.64**	.43	.35	.44	.57	.51	.52	.41					
20. G2 letter–word identification	.38	.47	.46	.39	.42	.40	.41	**.50**	.43	.48	**.55**	.46	.35	.43	.47	.48	.50	.39	.85				
21. G1 V-S working memory	.24	.22	.25	.19	.27	.28	.31	.25	.34	.31	.26	.33	.36	.31	.29	.27	.30	.26	.21	.15			
22. G2 V-S working memory	.14	.23	.20	.20	.32	.27	.31	.20	.40	.29	.16	.25	.30	.18	.20	.24	.22	.27	.24	.18	.24		
23. Sex	.02	−.13	−.12	−.09	.08	−.01	−.04	−.07	−.11	−.18	−.10	.06	−.03	−.03	.00	−.13	−.13	−.07	.09	−.11	−.01	−.06	
24. Language group	.05	.04	.13	.01	−.25	−.09	.01	−.08	.00	.07	−.08	.06	.05	.05	−.41	−.33	−.32	−.30	−.15	−.10	.00	−.08	−.06

Note. G1, Grade 1; G2, Grade 2; G3, Grade 3; G4, Grade 4; V-S, visual–spatial.
[a] Correlations are based on vertically equated scaled scores for this measure, due to differences in forms across grades, and for raw scores for all other measures.

Table 3
Correlations among mathematics, language, and control variables for native English speakers (above diagonal) and language minority learners (below diagonal), with strong correlations between language and mathematics measures bolded and underlined and with strong correlations between mathematics and control measures bolded ($N = 167$).

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
1. G1 arithmetic[a]		.54	.17	.26	.49	.58	.54	.53	.45	.55	.33	.36	.68	.55	.38	.36	.44	.34	.40	.21	.24	.08	–.03
2. G2 arithmetic[a]	.36		.51	.23	.32	.42	.45	.45	.45	.67	.45	.36	.37	.23	.23	.32	.17	.22	**.66**	**.58**	.21	.18	–.05
3. G3 arithmetic[a]	.23	.47		.53	.40	.36	.66	.44	.58	.60	.52	.40	.33	.24	.13	.34	.07	.28	**.53**	**.53**	.42	.16	–.04
4. G4 arithmetic[a]	.19	.66	.56		.45	.44	.56	.59	.62	.47	.64	.36	.34	.23	.06	.24	.16	.15	.45	.29	.08	.22	.20
5. G1 data analysis/probability	.47	.31	.09	.12		.44	.65	.51	.36	.56	.34	.50	.64	.34	**.52**	.45	**.54**	**.55**	**.50**	.36	.30	.36	–.05
6. G2 data analysis/probability	.42	.47	.26	.40	.34		.58	.71	.69	.67	.63	.60	.67	.58	.13	.32	.39	.45	.44	.37	.22	.33	.07
7. G3 data analysis/probability	.46	.53	.52	.40	.35	.49		.69	.64	.73	.53	.51	.62	.48	.32	**.52**	.47	**.55**	.43	.39	.42	.12	–.11
8. G4 data analysis/probability	.43	.55	.36	.56	.45	.57	.58		.74	.70	.76	.38	.63	.55	.27	.23	.39	.45	**.50**	.32	.35	.39	–.03
9. G2 algebra	.51	.65	.38	.53	.42	.61	.60	.56		.75	.68	.54	.53	.56	.17	.49	.17	.43	**.54**	.47	.39	.36	–.04
10. G3 algebra	.50	.48	.60	.46	.33	.48	.64	.46	.48		.62	.50	.60	.46	.23	**.52**	.22	**.52**	**.57**	**.54**	.36	.23	–.17
11. G4 algebra	.38	.55	.41	.63	.38	.56	.51	.71	.67	.57		.43	.48	.42	.21	.22	.11	.28	**.59**	.43	.38	.39	–.04
12. G2 geometry	.45	.44	.32	.34	.30	.48	.49	.64	.57	.32	.43		.58	.53	.36	.41	.35	.40	.44	.51	.23	.19	.11
13. G3 geometry	.46	.34	.30	.27	.33	.29	.57	.38	.57	.43	.46	.46		.67	.47	.30	.46	.41	.38	.24	.24	.23	.08
14. G4 geometry	.29	.46	.32	.45	.33	.51	.48	.60	.60	.42	.67	.57	.47		.48	.35	.28	.44	.35	.30	.48	.03	.13
15. G1 vocabulary	.34	.41	.35	.38	.43	.48	.48	**.50**	**.60**	**.53**	**.52**	**.57**	.46	**.61**		.59	.58	.59	.48	.32	.31	–.01	–.16
16. G2 vocabulary	.27	.49	.13	.30	.42	**.52**	.38	**.51**	**.58**	.40	**.51**	**.51**	.37	.44	.71		.35	.50	.45	.44	.26	.03	–.18
17. G1 oral comprehension	.32	.44	.23	.41	**.52**	.48	.40	**.64**	**.53**	.37	**.64**	**.55**	.41	**.60**	.70	.64		.50	.29	.23	.03	.06	–.37
18. G2 oral comprehension	.12	.41	.29	.37	.34	**.53**	**.50**	**.59**	**.52**	.45	**.51**	**.53**	.36	**.53**	.62	.62	.73		.44	.29	.44	.17	–.13
19. G1 letter–word identification	**.53**	.46	.42	.42	**.50**	.40	**.53**	**.61**	.46	.49	**.66**	.42	.37	.49	**.65**	**.56**	**.59**	.37		.84	.27	.34	–.01
20. G2 letter–word identification	.47	.41	.44	.42	.49	.43	.42	**.59**	.40	.39	**.62**	.44	.41	**.52**	**.59**	**.50**	**.62**	.44	.86		.12	.21	–.14
21. G1 V-S working memory	.21	.18	.15	.17	.16	.31	.19	.21	.32	.30	.25	.39	.35	.26	.33	.28	.36	.19	.22	.19		.22	.06
22. G2 V-S working memory	.16	.22	.25	.15	.22	.20	.35	.04	.41	.33	.09	.27	.36	.22	.25	.29	.09	.26	.10	.10	.25		.13
23. sex	–.04	–.14	–.21	–.22	.05	–.10	–.06	–.12	–.18	–.22	–.19	.04	–.07	–.14	.01	–.16	–.08	–.10	.08	–.11	.02	–.14	

Note. G1, Grade 1; G2, Grade 2; G3, Grade 3; G4, Grade 4; V-S, visual–spatial.

[a] Correlations are based on vertically equated scaled scores for this measure, due to differences in forms across grades, and for raw scores for all other measures.

comparisons with national norms as well as raw scores, which were used in subsequent analyses (with the exception of the arithmetic measure, for which vertically equated scaled scores were used to account for differences in forms across grades). As shown by the percentile ranks in the fourth column of Table 1, the sample was performing in the low-average range consistently over time on all of the mathematics measures: arithmetic (percentile ranks = 28.15–33.29), data analysis/probability (percentile ranks = 25.44–42.22), algebra (percentile ranks = 35.38–37.06), and geometry (percentile ranks = 24.64–34.15). The sample was also performing in the low to low-average range consistently over time on the language measures: vocabulary (percentile ranks = 22.20–22.93) and oral comprehension (percentile ranks = 28.04–32.25). The sample performed in the average range on the letter–word identification measure (percentile ranks = 60.14–65.53) but performed in the below-average range on the visual–spatial working memory measure (percentile ranks = 17.87–25.33).

Second, as shown on the right side of Table 1, we compared language minority learners and native English speakers on all measures. For the mathematics measures, we found that there were no significant differences on any of the measures at any time point with the exception of Grade 1 data analysis/probability, on which native English speakers significantly and moderately outperformed language minority learners ($d = -0.51$, $z = -3.04$, $p = .0024$). For both language measures, native English speakers significantly outperformed language minority learners at both time points: Grade 1 vocabulary ($d = -0.80$, $z = -4.58$, $p < .0001$), Grade 2 vocabulary ($d = -0.66$, $z = 3.82$, $p < .0001$), Grade 1 oral comprehension ($d = -0.65$, $z = -2.84$, $p = .0045$), and Grade 2 oral comprehension ($d = -0.59$, $z = -3.26$, $p = .0011$), as we would expect from previous research. For the control measures, there were no significant differences between language minority learners and native English speakers at Grade 1 or Grade 2 on the letter–word identification measure or on the visual–spatial working memory measure.

Third, we estimated correlations among all of the mathematics, language, and control variables, first for all students as shown in Table 2 and second separately for native English speakers (above the diagonal) and language minority learners (below the diagonal) as shown in Table 3. Strong correlations (based on Cohen's heuristic where *r*s at or above .50 are considered as strong) between language ability and mathematics measures are bolded and underlined, whereas strong correlations between mathematics and control measures are bolded. As shown by the bolded and underlined correlations, there were several strong concurrent and longitudinal bivariate relationships between language ability and mathematics measures, particularly for data analysis/probability, algebra, and geometry, a pattern that appeared to be somewhat more pronounced for language minority learners than for native English speakers (i.e., there were 24 strong language-with-mathematics correlations observed for language minority learners and 7 for native English speakers). There were no strong correlations observed between the arithmetic measure and the language measures for either language minority learners or native English speakers. In addition, as shown by the bolded correlations, mathematics had strong concurrent and longitudinal bivariate correlations with letter–word identification, highlighting the need to include this important confounding variable in subsequent models.

Fourth, we conducted exploratory analyses to investigate whether it would be appropriate to form a latent general language composite based on the vocabulary and oral comprehension indicators. Forming latent composites has the advantage of partialing out task-specific measurement error to create composites that are more reliable and represent theoretically motivated unobserved constructs (i.e., general language in this case) that can then be used in subsequent structural equation modeling (SEM) analyses (Brown, 2006). Forming a composite from these particular WJ-III measures also has an empirical basis in reading research, including evidence from confirmatory factor analyses with language minority learners (e.g., Lesaux, Crosson, Kieffer, & Pierce, 2010). In our sample, these two measures were strongly correlated in both Grade 1 ($r = .64$) and Grade 2 ($r = .60$) for all students as well as strongly correlated for the subsamples of language minority learners and native English speakers in each grade (see Table 3). In addition, in subsequent autoregressive structural equation models to address the first research question, the standardized factor loadings for this factor were appropriately high for the Grade 1composite (for vocabulary, standardized loading = .81–.85; for oral comprehension, standardized loading = .78–.83) and for the Grade 2 composite (for vocabulary, standardized loading = .81–.84; for oral comprehension, standardized loading = .74–.77).

Table 4
Results of regression models predicting gains in mathematical cognition presented in standardized regression coefficients with approximate *p* values based on bootstrapped confidence intervals ($N = 167$).

	Grade 4 arithmetic			Grade 4 data analysis/ probability		
Grade 1 language composite	0.34	0.33	0.14	0.68**	0.69**	0.67*
Grade 1 autoregressor (arithmetic or data analysis/probability)	0.14	0.13	0.07	0.10	0.10	0.14
Grade 1 visual–spatial working memory		0.05	0.09		−.01	−0.01
Grade 1 letter–word identification			0.27			−0.02
Sex	0.02	0.01	−0.02	−0.01	−0.01	−0.01
Language group	0.14	0.13	0.09	0.22*	0.23*	0.24
	Grade 4 algebra			Grade 4 geometry		
Grade 2 language composite	0.27	0.30	0.12	0.41**	0.42**	0.32*
Grade 2 autoregressor (algebra or geometry)	0.48**	0.54**	0.53**	0.32*	0.32*	0.31*
Grade 2 visual–spatial working memory		−0.17	−0.13		−.02	0.00
Grade 2 letter–word identification			0.27*			0.14
Sex	−0.03	−0.02	−0.02	0.00	−.01	0.00
Language group	0.02	0.03	−0.01	0.19*	0.19*	0.17

* $p < .05$.
** $p < .01$.

Does children's early language ability predict later gains in arithmetic, data analysis/probability, algebra, and geometry?

To shed light on the role of language in the development of mathematical cognition, we fitted four sets of autoregression models in an SEM framework using Mplus Version 4.2 (Muthén & Muthén, 2006). For each Grade 4 mathematics outcome, we investigated whether it was predicted by earlier language ability after controlling for the autoregressor (i.e., the same mathematics skills, measured in Grade 1 or Grade 2, depending on the measure) and the control measures of sex and language group status. We then tested whether earlier language ability predicted the Grade 4 mathematics outcome after controlling for visual–spatial working memory and letter–word identification, each entered sequentially. To test the statistical significance of the effect of language ability (and other measures) on each mathematical outcome, we used bootstrapping estimation with 5000 draws to estimate standard errors and 95% confidence intervals; this approach has several advantages over traditional approaches, particularly with small samples, primarily by avoiding the assumptions of asymptotically normal standard errors (e.g., Davison & Hinkley, 2008). The results of these four sets of autoregression models are displayed in Table 4 and described below.

As shown in Table 4, language ability predicted later gains in two of the four mathematics outcomes after accounting for the controls: data analysis/probability and geometry. For data analysis/probability, the effect of Grade 1 language ability was large in magnitude (standardized regression path = 0.67) and statistically significant, as indicated by the bootstrapped 95% confidence interval (*CI*) that did not cover 0 (unstandardized regression path = 1.32; bootstrapped *CI* = 0.08 to 2.66).[1] For geometry, the effect of language ability was moderate in magnitude (standardized regression path = 0.32) and statistically significant, as indicated by the bootstrapped 95% confidence interval that did not cover 0 (unstandardized regression path = 0.53; bootstrapped *CI* = 0.03 to 1.20).[2] Each of these two effects was significant with or without controlling for visual–spatial working memory and

[1] For data analysis/probability, the absolute goodness of fit of the full model presented in Table 4 was not ideal ($\chi^2 = 14.59$, $df = 5$, $p = .0123$, root mean square error of approximation [*RMSEA*] = .11, comparative fit index [*CFI*] = .86, Tucker–Lewis index [*TLI*] = .48), perhaps due to the inclusion of several nonsignificant paths between the control variables and the outcome. However, our research questions focused not on the viability of an overall model but rather on the significance of the specific regression path between oral language and data analysis/probability (or, viewed differently, the comparison between models with and without this path of interest).

[2] For geometry, the absolute goodness of fit for the full model presented in Table 4 was excellent ($\chi^2 = 4.79$, $df = 5$, $p = .4422$, *RMSEA* < .00, *CFI* = 1.00, *TLI* = 1.01).

letter–word identification. By contrast, language ability did not significantly predict arithmetic (unstandardized regression path = 0.22; bootstrapped *CI* = −0.76 to 1.33) or algebra (unstandardized regression path = 0.20; bootstrapped *CI* = −0.44 to 0.80). Each of these two effects was not significant whether or not we controlled for visual–spatial working memory and letter–word identification.

Do the relations between language ability and gains in mathematical cognition differ between language minority learners and native English speakers?

To address our second research question—whether the relations between language ability and gains in the four domains of mathematical cognition differ between language minority learners and native English speakers—we fitted a series of multiple-group autoregressive structural equation models in which language group was specified as the group. These models were based on the same models reported above but were fitted simultaneously to the subsets of data for language minority learners and native English speakers. For each mathematics outcome, in the baseline model all structural parameters were fixed to be equal across groups, whereas latent factor means, intercepts, residual variances, and residual covariances were allowed to differ across groups, as is common practice (Brown, 2006). We then used a likelihood ratio test to compare the goodness of fit of this baseline model with a model in which the effect of language ability on the given Grade 4 mathematics outcome was allowed to differ between language minority learners and native English speakers. To check these results, we also fitted single-group models in which we included an interaction between language group and early oral language.

Results indicated the effect of early language ability on gains in mathematical cognition did not differ between language minority learners and native English speakers. Allowing this path to vary by language group did not significantly improve the goodness of fit of the models for any of the outcomes: arithmetic ($\Delta\chi^2 = 1.791$, $df = 1$, $p = .1808$), algebra ($\Delta\chi^2 = 2.998$, $df = 1$, $p = .0834$), data analysis/probability ($\Delta\chi^2 = 0.003$, $df = 1$, $p = .9563$), and geometry ($\Delta\chi^2 = 1.213$, $df = 1$, $p = .2707$). There was a trend such that point estimates for the standardized effect of language ability on gains in mathematics were (nonsignificantly) higher for the language minority learners than for the native English speakers across all four outcomes. These differences were small (differences in standardized regression paths = 0.11–0.28), so they may have been statistically significant if we had larger subsamples of language minority learners and native English speakers. Additional single-group models indicated that an interaction between language group and early oral language was not a significant predictor for any of the mathematics outcomes.

Discussion

The primary goal of this study was to explore the linguistic basis of mathematics. Although language is both implicitly and explicitly involved in the teaching and learning of mathematics, the little research in this area has focused on the role of language in the development of number (e.g., Le Corre & Carey, 2007; Libertus et al., 2011; Sarnecka & Carey, 2008) or on the relation between phonological processing and arithmetic (e.g., Hecht et al., 2001; Simmons & Singleton, 2008). Furthermore, this research has mostly been conducted with native English-speaking samples. Building on this research, we explored how language ability more broadly defined related to mathematical development in a linguistically diverse sample of children from first to fourth grades. Overall, our results suggest that language ability does indeed play a role in children's mathematical development, but only for some aspects of mathematical cognition. Below we discuss two major findings that emerged from this study.

Language matters for mathematical meaning making

First, our results showed that language ability shared several strong concurrent and longitudinal relations with data analysis/probability, algebra, and geometry but not with arithmetic. When accounting for autoregressive effects and control variables, however, language ability predicted later gains only in data analysis/probability and geometry. These results suggest that for the sample studied, language ability is not directly involved in learning how to manipulate quantities and execute algorithms—as is the case with arithmetic and algebra—but is involved in how children learn to make meaning of mathematical content.

Arithmetic and algebra have in common that both use the same algorithms and rules of operation to solve mathematical statements, with the difference being that arithmetic statements involve known quantities, whereas algebraic statements contain unknown values (i.e., variables). That language ability did not predict gains in either arithmetic or algebra, therefore, suggests that language ability might not be involved in learning how to manipulate precise numerical quantities according to standard algorithms and procedures. This is consistent with previous research suggesting that arithmetic and algebra share a common origin. Specifically, arithmetic and algebraic reasoning have been shown to draw on the same cognitive resources (Fuchs et al., 2012), and arithmetic skills have been found to provide the foundation for algebraic cognition (Fuchs et al., 2012; National Mathematics Advisory Panel, 2008; Pillay, Wilss, & Boulton-Lewis, 1998).

However, our finding contrasts with research suggesting a link between language and arithmetic performance (e.g., Dehaene et al., 1999; LeFevre et al., 2010; Spelke & Tsivkin, 2001). Our measure of arithmetic required children to solve procedural problems with the four operations that often required regrouping, whereas previous studies have tended to focus only on addition and subtraction problems memorized by rote. In a study with bilingual adults, Spelke and Tsivkin (2001) found that exact multiplication facts were retrieved more efficiently in the language of training versus the untrained language, suggesting that adults might use language-dependent representations of exact number to store multiplication problems by rote. Our results are not consistent with this conclusion, at least for children solving complex arithmetic problems that go beyond rote memorization of number facts. Thus, it may be the case that general language ability matters for addition and subtraction, but the role of language in arithmetical multiplication and division remains to be determined. In any case, our findings suggest that general language ability is less important in learning manipulations—either with Arabic symbols as in arithmetic or with abstract symbols as in algebraic reasoning—meaning that other cognitive systems beyond language are likely involved in how exact numbers are manipulated separate from how exact numbers are cognitively represented (Jordan & Levine, 2009; Jordan, Mulhern, & Wylie, 2009; Locuniak & Jordan, 2008).

We further interpret our findings to indicate that, while playing a limited role in numerical manipulations, language ability is necessary for making meaning of mathematical content. In other words, language ability may be fundamental in forming mathematical concepts and representations. Both data analysis/probability and geometry require conceptual understanding of mathematical relations that is not directly dependent on manipulating exact numbers through procedures or algorithms. That is, these tasks can be completed only if children make sense of the mathematical concepts and representations as opposed to blindly relying on algorithms to solve mathematical problems. That language ability predicted gains in data analysis/probability and geometry even after controlling for autoregressive effects and control variables suggests that language ability is central to the development of higher order mathematical concepts, including those that do not appear to have heavy language demands. This is in contrast to the hypothesis that higher order forms of mathematics are *less* dependent on language (Dehaene et al., 1999). Our results do not rule out the effect of very basic numerical competencies such as the potentially inherent approximate number system on later mathematical development (Halberda, Mazzocco, & Feigenson, 2008; Libertus & Brannon, 2010; Libertus et al., 2011), but our results do suggest a prominent role for early language competencies, consistent with the results of LeFevre and colleagues (2010).

Language matters for mathematics regardless of language background

Second, we found the relation between language ability and mathematical cognition to be generally similar for both language minority learners and their native English speaking peers studied, suggesting that early language experiences are important for later mathematical development regardless of language background. These results converge with those of Kleemans and colleagues (2011), who also found no differences between native Dutch speakers and second language Dutch learners in how domain-general skills assessed in Dutch (i.e., grammatical ability, phonological awareness, and working memory) accounted for early numeracy in 6-year-olds. Thus, our findings are consistent with the growing body of research indicating that the factors that put children at risk for academic difficulties in high-poverty urban settings are the same for both language minority learners and their native English-speaking classmates (e.g., Abedi & Lord, 2001; Carlisle, Cortina, & Zeng, 2010; Lager, 2006), implicating a general need for opportunities to learn the language and cognitive functions associated with mathematical success over individual native language background. In other words, specific to the results of this study, it is clear that both language minority learners and their peers growing up in high-poverty settings need intensive and targeted opportunities to develop the language and skills associated with mathematical concepts and representations.

At the same time, however, we identified a somewhat more pronounced relation between language and mathematical cognition for language minority learners (i.e., a greater number of strong significant correlations for language minority learners and a trend toward significance in the SEM analyses), indicating that although language is important for both groups, the impact may be greater for language minority learners. Thus, although language minority learners are vulnerable to the same risk factors associated with academic difficulties as their native English-speaking peers, one risk factor is unique to this population: These students enter schools faced with the challenge of simultaneously learning academic content *and* developing proficiency in the language of instruction. This suggests that language minority learners are in particular need of rich opportunities to learn language during the early to middle childhood years in order to develop age-appropriate mathematical proficiency. Given the increasing importance of mathematical proficiency to compete in a global market and respond to global challenges (Peterson, Woessmann, Hanushek, & Lastra-Anadón, 2011), it is critical for educators to minimize the challenges language minority learners face not only in acquiring language-dependent mathematical concepts and representations that are challenging for all students with underdeveloped language skills but also in developing the language proficiency to profit from classroom instruction. For instance, students need language- and content-based instruction with a focus on teaching both specialized vocabulary (and the often abstract concepts that such words represent) and the specialized structures of language in academic speech and text—often referred to as elements of *academic language*. Such language is an essential tool for reading, writing, and critical thinking and one that presents a particular source of difficulty for many students, especially language minority students, impeding achievement in all academic areas, including mathematics.

Limitations and future directions

Although this study contributes to a growing body of studies investigating the role of language in children's mathematical cognition, the findings also invoke several questions and limitations to be addressed in subsequent research. First, we controlled for some, but not all, skills that can be expected to have an influence on children's mathematical development, most notably the approximate number system (e.g., Halberda et al., 2008). Even though LeFevre and colleagues (2010) found that linguistic skills were more important than either visual–spatial working memory or preverbal number skills in predicting mathematical outcomes, more research is needed to determine how various skills and processes interact with language ability to explain individual differences in children's mathematical development. In particular, research should investigate how language ability and the preverbal number system uniquely contribute to growth in various mathematical abilities. It is also important to study how mathematical language influences mathematical cognition separately from general language ability. That is, children with better overall language ability might also have more mathematical language, and it could be that mathematical language is the stronger predictor of mathematical development. In addition, given that the children in this study had been exposed to multiple years of formal mathematics instruction, we must generate an understanding of the role of instruction in facilitating the coordination among the preverbal number system, the verbal number system, and mathematical performance.

Our findings raise the issue of whether language minority learners would have performed better if they had been assessed in their native language. Dehaene and colleagues (1999) and Spelke and Tsivkin (2001) found not only that arithmetic facts are stored in language-specific formats but also that there is a cost (in time) associated with retrieving arithmetic facts in a language other than the one in which the facts were learned. Thus, one possibility is that language minority learners in our study might have had the mathematical knowledge but were not able to easily access the information in English. Although we were not able to assess language minority learners in their first language due to practical constraints, availability of appropriate measures, and the trade-offs in light of the overall aims of the research project, it is of considerable interest to determine how these findings might differ when investigating the relation between native language ability and mathematical performance demonstrated in both native and second languages. It is worth noting that language minority learners did not statistically outperform native English speakers on the paper-and-pencil computation task that had relatively few verbal demands, suggesting that language minority learners were not especially disadvantaged by being assessed in English. However, future research is warranted.

Finally, it is worth noting that the moderate sample size yielded relatively limited statistical power to detect small effects, so although the null results comparing language minority learners and their native English-speaking classmates may indeed reflect the growing body of research in the urban setting that demonstrates similarities in the role of predictors of academic performance across these groups (Carlisle et al., 2010; Lesaux & Kieffer, 2010), comparative research with larger samples is needed.

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